52

Chapter

Product and System Reliability

Chapter Outcomes

After completing the chapter, you will be able to:

- Identify failure causes and define failure rate.
- Define reliability.
- Derive reliability function.
- Describe reliability functions of well known distributions.
- Evaluate reliability of system when components are in series, parallel and combination of series parallel systems.
- Describe the various ways to test reliability.

52.1 INTRODUCTION

Customers always want that the products that are purchased should have a long service life and during this life it should give intended service and utility with few failures. As product becomes more complex, the problems of failures will increase over time. The improvement in effectiveness of such complex systems has therefore acquired special importance in recent years. The effectiveness of a system is its suitability for the fulfilment of the intended function and the efficiency of utilizing the means put in to it. The suitability of performing definite task is primarily determined by the reliability and quality of the system. An evaluation of a system reliability becomes essential to decide whether a system will accomplish its mission successfully.

During the past, very high safety factors were introduced which added tremendously to its weight as well as the cost and another approach was to learn from the previous mistakes (failures) of previous designs. These approaches become impractical for the design of new products and systems where each design is different from other and each design has to be right first time and all the time. There is no much time for trial and error. Reliability is to be built in to the product right at the design stage and quality and hence the reliability should be built during subsequent stages of manufacture.

Reliability engineering is a emerging area, which is a collection of many tools. Reliability engineering is a combination of management and technical disciplines, which has a specific objective — that is the assurance of intended performance for a specified time span of products. It is a formalized approach to achieve optimum product reliability using management, engineering, mathematical and statistical elements and concepts. A high level of reliability can be achieved only through an integrated effort, which ensures that there are no weak links in the total process of design, development and manufacturing.

52.2 DEFINITION OF RELIABILITY

"Reliability is the ability of an item to perform its intended function under stated operation conditions for a given period of time."

The definition stresses on four significant elements:

1. Probability
2. Intended Function
3. Time
4. Operating Conditions.

1. Probability

Consideration of variation makes reliability a probability. It is possible to identify the frequency distribution of an item, which permits prediction of life of the item. *e.g.,* the probability of an item functioning is 0.85 for 60 hours indicates that only 85 times out of 100, we would expect the item to be functioning for a period of 60 hours.

2. Intended Function

For an item to be reliable, it must perform a certain functions satisfactory when called upon to do, while considering the reliability of an item, the criteria of what is considered as the required function have to be exactly spelled out in advance. Thus, criteria must be established in all cases, which clearly specify and define what is considered as intended function.

3. Time

Time is the most important factor in the assessment of reliability, since it represents a measure of the period during which one can expect a certain degree of performance from an item.

4. Stated Conditions

The application and operating circumstances under which an item is put to use is an important component of reliability. As the operating conditions, change the reliability of an item also changes. Operating conditions such as temperature, humidity, torque, and corrosive atmosphere all have a definite effect on performance.

Thus, reliability can be stated as follows:

"The reliability of a 60 watts incandescent bulb has been estimated to be 0.95 for 1200 hours providing 20 candles output under 180-230 volts and at normal environmental conditions."

52.3 FAILURE

Failure of an item represents unreliability. Thus, to compute the reliability of an item, it is necessary to understand the concept of failure. A deviation in the properties of an item from the prescribed conditions is considered as fault. A state of the fault is denoted as "Failure."

An item is considered to have failed under one of the following conditions:

1. When it becomes completely inoperable.
2. When it is still operable, but no longer able to perform a required function.
3. When a serious deterioration makes the item unsafe for its continued use.

52.3.1 Causes of Failures

There are many specific causes of failures of components and systems. Due to the complexity of the system, some are known and some are unknown. Some of the causes of failures are:

- Deficiencies in design.
- Improper selection of process and manufacturing technique.
- Lack of knowledge and experience.
- Errors of assembly.
- Improper service conditions.
- Inadequate maintenance.
- Variation in environmental and operating conditions.
- Human errors.

52.3.2 Nature of Failures

An item may fail in many ways. An understanding of these failures help in taking appropriate corrective measures for achieving better reliability. The different modes of failure are:

1. **Catastrophic Failures:** In this case, a normally operating item suddenly becomes inoperative. Example: Blowing of a fuse or electric bulb.
2. **Degradation (Creeping) Failures:** These failures occur gradually because of change in some parameter with time. Example, change in resistance will affect the performance of a resistor.
3. **Independent Failures:** These are the failures, which occur independently and does not depend on failure of the other.
4. **Secondary Failures:** A secondary failure occurs as a result of some primary failure.
5. **Failure due to Improper Handling and Misuse:** *e.g.*, overloading (stressing beyond the capacity).

52.3.3 Phases of Failures (Bath Tub Curve)

Analysis of failure data has shown that failures in general can be grouped in to different modes depending upon the nature of the failure. When a large number of units are put into operation, it is likely that there is large number of failures initially. These failures are called Initial Failures or Infant Mortality. After the initial failures, for a long period of time of operation fewer failures are reported but it is difficult to determine their cause. The failures during this period are often called random failures or catastrophic failures. This is the period of normal operation. As the time passes, the units get worn out due to wear and tear and begin to deteriorate. Here in this period, the failures are due to wear and tear and due to ageing. This region is called the wear out region.

These three phases of failures are represented in Fig. 52.1 and the characteristics of each phase are shown in table 52.1.

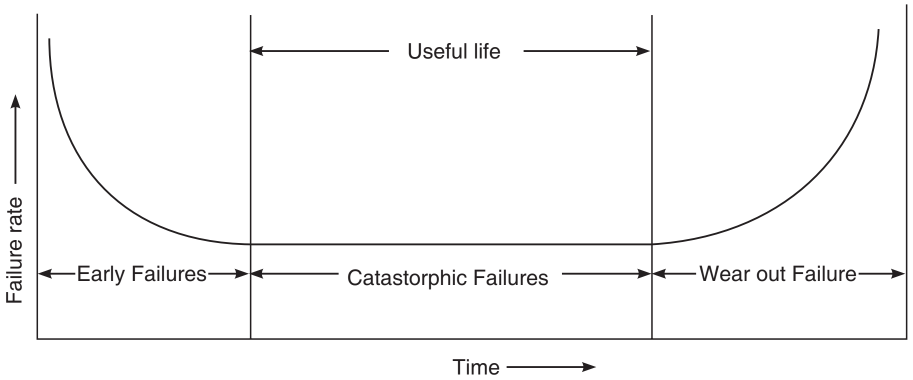


Fig. 52.1: Bath Tub Curve.

Table 52.1: Characteristics of Various Phases of Bath Tub Curve

1. **Early Failures**
   - These failures occur at the beginning due to the probability of defective design, manufacturing or assembly and quality control techniques during manufacturing.
   - These are eliminated by debugging or burn in process. The weak and substandard products/components that fail during early hours of system operation are replaced by good or tested components. Debugging is a method of accelerating the completion of early failures by operating the system continuously for number of hours, correcting them and then releasing the system for actual use.
   - Debugging is done generally prior to dispatch to the user to ensure the detection and elimination of early failures.
   - Warranty is based on the concept of early failures.
2. **Catastrophic (Chance) Failures**

   These failures are predominant during actual working of the system. They occur randomly and unexpectedly. The failure rate is fairly constant. These are caused due to sudden stress accumulation beyond the design strength of the material. This phase is called the useful life of the component. The failures at this stage can be minimised by introducing redundancy in the system.
3. **Wear Out Failures**

   The item is more likely to fail due to wear and tear and the number of failures will be high. This is a typical ageing problem. Proper care and maintenance will reduce the failures at this stage.

52.4 MEASURES OF RELIABILITY

1. Failure Rate

Failure rate is expressed in terms of failures per unit time, *i.e.,* as failures per hour, or failures per 100 or 1000 hours. Failure rate is the ratio of number of failures (*f*) during a specified test interval to the total test time of items undergoing test.

$$\lambda = \frac{f}{T}$$

λ = Failure rate

f = Number of failures during the test interval.

T = Total test time.

When the design is new, failure rate is high and when the design is matured, the failure rate is fairly constant. Smaller the value of failure rate, higher is the reliability of the system.

2. Mean Time Between Failures (MTBF)

MTBF is referred to as the average time of satisfactory operation of the system. Larger the MTBF, higher is the reliability of the system. It is applicable to repairable systems and is expressed in hours, *e.g.* If an item fails 8 times over a period of 40,000 hours of operation the MTBF would be 500 hrs. During the operating period, the failure rate is fairly constant MTBF is the reciprocal of the constant failure rate or the ratio of test time to number of failures.

3. Mean Time To Failure (MTTF)

This is applicable to non-repair systems. The mean time to failure is expressed as the average time an item is expected to function before failure. If we have the life test information on 'n' items with failure times t_1, t_2, ... t_n, then the mean time to failure is defined as

$$\text{MTTF} = \frac{1}{n}\sum_{i=1}^{n} t_i$$

52.5 RELIABILITY FUNCTION DERIVATION

Reliability is defined as the probability that a system (component) will function over some period of time 't'. To express this relationship mathematically, we define the continous randon variable 'T' to be the time to failure of the system (or component), $T \geq 0$.

Then, reliability can be expressed as

$$R(t) = \text{Prob}\{T \geq t\}$$

Where $R(T) \geq R(0) = 1$ and $\lim_{t \to \infty} R(t) = 0$ ... (1)

For a given value of t, $R(t)$ is the probability that the time to failure is greater than or equal to 't'.

We define,

$$F(t) = 1 - R(t) = \text{Prob}\{T < t\}$$

Where $F(0) = 0$,

and $\lim_{t \to \infty} F(t) = 1$ ... (2)

then, $F(t)$ is the probability that a failure occurs before time 't'. Thus, $R(t)$ is the reliability function and $F(t)$ is the cumulative distribution function (CDF) of the failure distribution. A third function called probability density function (PDF) is defined as

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} \qquad \text{... (3)}$$

This function describes the shape of the failure distribution. The three functions are illustrated in the Fig. 52.2.

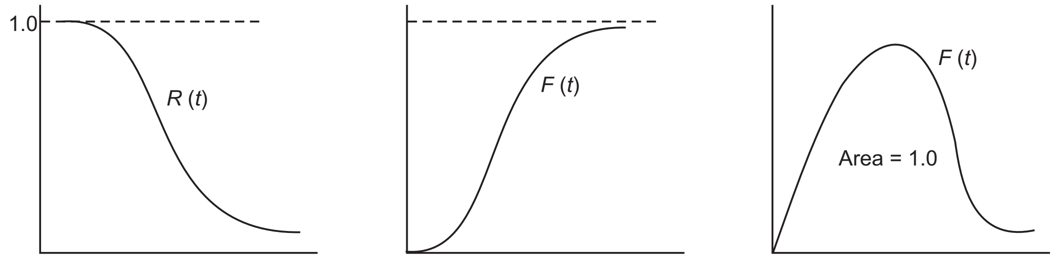


Fig. 52.2: (a) Reliability function (b) Cumulative distribution function (c) Prob. density function.

The probability density function (*pdf*), has the following two properties

$$f(t) \geq 0 \text{ and } \int_0^{\infty} f(t)dt = 1$$

Given *Pdf* . $f(t)$, then

$$F(t) = \int_0^t f(t')dt'$$

$$R(t) = \int_t^{\infty} f(t')dt'$$

Both reliability function $R(t)$, and the cumulative density function represent areas under the curve defined by $F(t)$. Since the area under the curve is equal to one, both the reliability and failure probability will be defined so that,

$$O \leq R(t) \leq 1 \text{ and } O \leq F(t) \leq 1$$

The function $R(t)$ is normally used when reliabilities are being computed and the function $F(t)$ is normally used when the failure probabilities are being computed. The graphical representation of *pdf* [$f(t)$] provides a visual representation of the failure distribution.

Component Reliability From Test Data

Consider a set of 'N' components in operation from time $t = 0$. With the progress of the time, the components fail. After a certain period t, consider that number of components surviving are n and the number of components that have failed are m.

The components fail independently with the probability of failure.

$\therefore$ $N = (n + m)$ is constant through out the test. This is because, as the test proceeds, the number of failed components 'm' increases as the number of surviving components (n) decreases.

Probability of survival (reliability) is expressed as the fraction at any time 't' during the test and is given by

$$R(t) = \frac{n}{N} = \frac{n}{m+n} \quad \text{... (1)}$$

The probability of failure or unreliability is expressed at any time 't' can be expressed as

$$F(t) = \frac{m}{m+n} \quad \text{... (2)}$$

At any time t,

$$R(t) + F(t) = 1 \quad \text{... (3)}$$

Because $R(t)$ and $F(t)$ are mutually exclusive events equation (1) can be rewritten as

$$R(t) = \frac{n}{n+m} = \frac{N-m}{N} = \left(1 - \frac{m}{N}\right) \quad \text{... (4)}$$

Differentiating the equation (4) w.r.t. to 't', we have,

$$\frac{d\,R(t)}{dt} = \frac{d}{dt}\left(1 - \frac{m}{N}\right) = -\frac{1}{N}\frac{dm}{dt} \quad \text{... (5)}$$

Rearranging the terms in the equations (5),

$$\frac{dm}{dt} = -N\frac{d\,R(t)}{dt} \quad \text{... (6)}$$

This equation (6) represents the rate at which the components fail since $m = (N - n)$, on differentiation.

$$\frac{dm}{dt} = \frac{d}{dt}(N-n)$$

$$= -\frac{dn}{dt} \quad \text{... (7)}$$

This is the negative rate at which the components survive. The term $\frac{dm}{dt}$ can be interpreted as — number of components failing in the time interval. 'dt' between the times t and $t + dt$ which is equivalent to the rate at which the components still in the test at time 't' is failing.

Dividing equation (6) by 'n' on both sides

$$\frac{1}{n}\frac{dm}{dt} = -\frac{N}{n}\frac{d\,R(t)}{dt} \quad \text{... (8)}$$

The L.H.S. of equation is the failure rate (λt) and $\frac{n}{N} = R(t)$.

Thus,
$$\lambda(t) = -\frac{1}{R(t)}\frac{d\,R(t)}{dt} \quad \text{... (9)}$$

Relation Between $f(t)$, $R(t)$ and $\lambda(t)$

1. Probability Density Function [$f(t)$]

It is the probability that a random trial yields the value of 't' within the interval from t_1 to t_2 and expressed as

$$\int_{t_1}^{t_2} f(t)\,dt$$

$f(t)$ is the density function for a continuous random variable.

2. Distribution Function [$F(t)$]

It is the probability that in a random trial, the random variable is not greater than 't'.

$$\therefore \quad F(t) = \int_{-\infty}^{t} f(t)\,dt$$

$F(t)$ is recognized as unreliability function.

3. Reliability [$R(t)$]

It expresses the probability that the variable is at least as large as 't'.

$$R(t) = \int_{t}^{\infty} f(t)\,dt$$

and
$$R(t) = 1 - F(t).$$

4. Failure Rate [λ. (*t*)]

The rate at which failures occur in the interval t_1 and t_2 is called the failure rate. It is expressed as the conditional probability that failures occur in the interval t_1 and t_2, given that failures have not occurred prior to t_1, *i.e.*, the start of the interval.

Failure rate is given by

$$\lambda(t) = \frac{\int_{t_1}^{t_2} f(t)\,dt}{(t_2 - t_1)\int_{t_1}^{\infty} f(t)\,dt}$$

$$= \frac{\int_{t_1}^{\infty} f(t)\,dt - \int_{t_2}^{\infty} f(t)\,dt}{(t_2 - t_1)\int_{t_1}^{\infty} f(t)\,dt}$$

$$= \frac{R(t_1) - R(t_2)}{R(t_1)(t_2 - t_1)}$$

If we substitute $t_1 = t$ and $t_2 = t + \Delta t$

The failure rate can be expressed as

$$\lambda(t) = \frac{R(t) - R(t + \Delta t)}{\Delta t\, R(t)}$$

As a special case,

where $f(t)$, the probability density function is exponential the various functions can be computed as

Probability density function $f(t)$ is

$$f(t) = (\lambda e^{-\lambda t})$$

where λ is the constant failure rate.

$$f(t) = \int_0^t \lambda e^{-\lambda t}\,dt$$

$$= 1 - e^{-\lambda t}$$

Reliability function can be calculated as

$$R(t) = \int_t^{\infty} \lambda e^{-\lambda t}\,dt$$

$$= e^{-\lambda t}$$

$$R(t) = 1 - F(t)$$

52.6 HAZARD RATE Z (*t*)

Hazard rate or instantaneous failure rate is defined as the limit of failure rate as the time interval length approaches to zero. It is a measure of instantaneous speed of failure.

It is expressed as

$$Z(t) = \lim_{\Delta t \to 0}\left[\frac{R(t) - R(t + \Delta t)}{\Delta t\, R(t)}\right]$$

$$= \frac{-1}{R(t)}\frac{d(Rt)}{dt}$$

$$\therefore \qquad Z(t) = \frac{f(t)}{R(t)}$$

$$\text{as} \qquad f(t) = \frac{-d\,R(t)}{dt}$$

Relationship between $R(t)$ and $Z(t)$

$$Z(t) = \frac{F(t)}{R(t)} = \frac{d\,F(t)}{dt} \cdot \frac{1}{R(t)}$$

Integrating the above equation between 0 to t

$$\int_0^t Z(t)\,dt = \int_0^t \frac{d\,F(t)}{dt}\frac{1}{R(t)}$$

$$= \int_0^t \frac{d\,F(t)}{1-F(t)} = -\log\left[1-F(t)\right]_0^t$$

$$= \left[\log R(t)\right]^t$$

$$= -\log R(t) \text{ as } R(0) \text{ and } \log R(0) = 0$$

$$\therefore \qquad \boxed{R(t) = e^{-\int_0^t Z(t).dt}}$$

Cumulative Distribution Function

$$F(t) = 1 - R(t) = 1 - e^{-\int_0^t Z(t).dt}$$

Probability density function

$$F(t) = Z(t)\,.\,e^{-\int_0^t Z(t).dt}$$

Rearranging the equation and integrating with proper limits, we have,

$$\lambda(t).dt = \frac{-d\,R(t)}{R(t)}$$

$$\text{or} \qquad \int_0^t \lambda(t)\,dt = -\int_0^R \frac{d\,R(t)}{dt} = -\ln R(t) \qquad \text{... (10)}$$

$$\therefore \qquad \ln R(t) = -\int_0^t \lambda(t).dt \qquad \text{... (11)}$$

Initially, at $t = 0$, $R(t) = 1$, we obtain

$$R(t) = \exp\left[-\int_0^t \lambda(t)\,dt\right] \qquad \text{... (12)}$$

This is (equation 12) is a general formula for computing reliability $\lambda(t)$ can be any variable and integratable function of time. But, if we specify that $\lambda(t)$ is constant overtime and $\lambda(t) = \lambda$ (say).

The reliability formula becomes

$$R(t) = e^{-\lambda t}$$

Properties

The probability of survival or reliability $R(t)$ at time 't' has the following properties.

1. $0 \le R(t) \le 1$

2. $R(0) = 1$ and $R(\infty) = 0$

As special case, when the probability function is exponential and failure rate is constant.

Then
$$Z(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

When the failure rate is constant, hazard rate is also constant and is equal to the failure rate.

Mean Time To Failure (MTTF)

The Mean Time To Failure will assume to be the same for all the components, which are identical in the design and operate under the conditions, which are identical.

MTTF is given by the mathematical expectation of the random variable 'T' describing the MTTF of the component.

Therefore,
$$\text{MTTF} = E(T) = \int_0^\infty f(t)\,dt$$

This is the mean or expected value of the probability distribution defined by $F(t)$.

We have the probability density function (PDF)

$$F(t) = \frac{d.f(t)}{dt} = \frac{-d\,R(t)}{dt}$$

$$\text{MTTF} = \int_0^\infty \frac{-d\,R(t)}{dt} t.dt$$

Using integration by parts

$$\text{MTTF} = -t.R(t)\Big|_0^\infty + \int_0^\infty R(t)\,dt$$

$$= \int_0^\infty R(t)\,dt$$

Since,
$$\lim_{x\to\infty} t.R(t) = \lim_{x\to\infty} t.\exp\left[-\int_0^t \lambda(t^1).dt^1 = 0\right]$$

and $R(0) = 0$.

Constant Hazard Model

The constant hazard model can be expressed as

$$Z(t) = \lambda$$

where λ is constant and independent of time.

An item with constant hazard rate will have the following reliability and associated functions.

$$f(t) = \lambda.\,e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$

The mean time to failure of the item is

$$\text{MTTF} = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda}$$

52.7 RELIABILITY AND HAZARD FUNCTIONS FOR WELL KNOWN DISTRIBUTIONS

52.7.1 Exponential Distribution

Exponential distribution is widely used in reliability. A constant failure rate model for continuously operating system leads to an exponential distribution. Replacing a time dependent failure rate $\lambda(t)$ by a constant λ in the *pdf* equation.

We have

$$F(t) = \lambda . e^{-\lambda t}$$

Similarly, CDF becomes

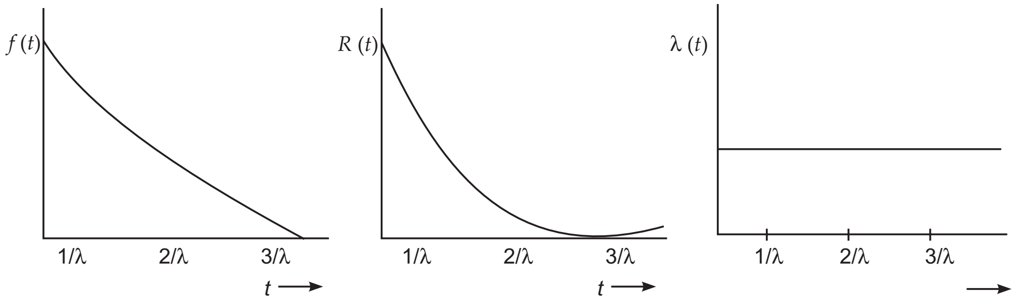
$$F(t) = 1 - e^{-\lambda t}$$

and the reliability can be written as

$$R(t) = e^{-\lambda t}$$

$$\text{MTTF} = \frac{1}{\lambda}$$

The exponential failure density function is represented in Fig. 52.3.



(*a*) Time to failure pdf (*b*) Reliability (*c*) Failure rate

Fig. 52.3: The exponential Distribution.

A device described by a constant failure rate and therefore by an exponential distribution of times to failure has the following property of "Memorylessness." The probability that it will fail during same period in the future is independent of its age.

52.7.2 Normal Distribution

The normal distribution takes the well-known bell-shape and to describe the time dependence of reliability problems. The pdf for normal distribution is given by the following equation with '*t*' as a random variable.

$$F(t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]$$

where μ is here is the MTTF.

$$F(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] dt$$

In the standardized normal form,

$$F(t) = \phi\left[\frac{t-\mu}{\sigma}\right]$$

and Reliability for normal distribution is given by

$$R(t) = 1 - \phi\left[\frac{t-\mu}{\sigma}\right]$$

and the failure rate is obtained by the equation,

$$\lambda(t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2 \left(1-\phi\frac{t-\mu}{\sigma}\right)\right]^{-1}$$

The reliability and *pdf* are plotted for times to failure as shown in Fig. 52.4.

As indicated by the behaviour of failure rate, normal distributions are used to describe the reliability of the equipment to situations other than to which constant failure rates are applicable. It is useful in describing the reliability in situations in which there is a reasonably well-defined wear out time μ.

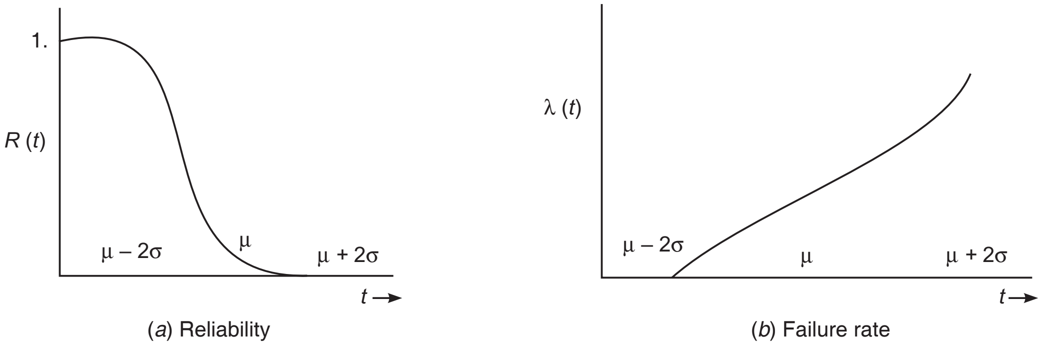


Fig. 52.4: Normal Distribution.

52.7.3 Log Normal Distributions

The log normal is related distribution that has been found to be useful in describing failure distributions for a variety of situations especially when the time to failure is associated with large uncertainty. The pdf for the time to failure is given 6y

$$f(t) = \frac{1}{\sqrt{2\pi}St} \exp\left\{-\frac{1}{2S^2}\left[\ln\frac{t}{t_0}\right]^2\right\}$$

corresponding CDF is expressed as

$$F(t) = \phi\left[\frac{1}{S}\ln\frac{t}{t_0}\right]$$

however to is not the MTTF.

$$\text{MTTF} = \mu = \text{to} \,.\, \exp(S^2/2)$$

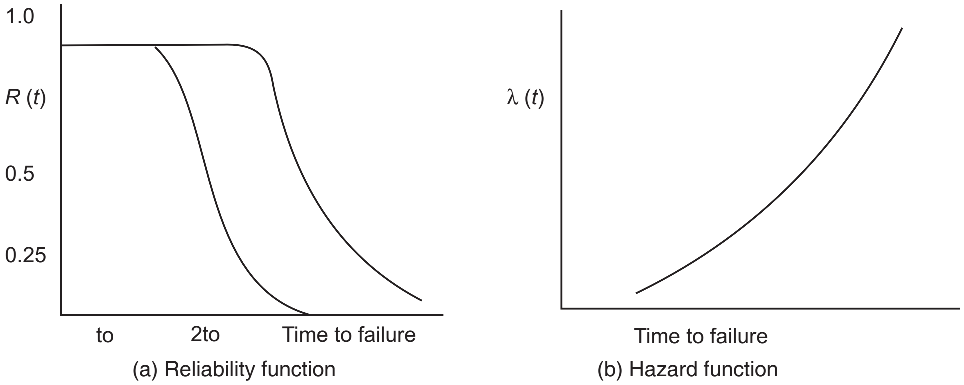


Fig. 52.5: Log normal distribution.

The log normal distribution is frequently used to describe fatigue and other phenomenon that are caused by ageing or wear and that result in failure rates that increase with time.

The log normal reliability function and hazard functions are shown in Fig. 52.5. The failure can be increasing or decreasing depending on value of '*S*'.

52.7.4 Weibull Distribution

One of the most useful probability distribution in reality is the weibull. Weibull failure distribution may be used to model both increasing and decreasing failure rates. It is characterised by the hazard rate function of the form.

$$\lambda(t) = a \,.\, t^b$$

which is a power function.

The function $\lambda(t)$ is increasing for $a > 0$, $b > 0$ and is decreasing for $a > 0$, $b < 0$. For convenience, $\lambda(t)$ can be expressed as

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \quad \theta > 0, \beta > 0, t \geq 0$$

$R(t)$ is expressed as

$$R(t) = \exp\left[-\int_0^t \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} .dt\right]$$

and

$$f(t) = -\frac{dR(t)}{dt} = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{\beta-1} .e^{-(t/\theta)\beta}$$

Beta (β) is referred to as the shape parameter. Its effect on the distribution can be represented in Fig. 52.6 for several different values β.

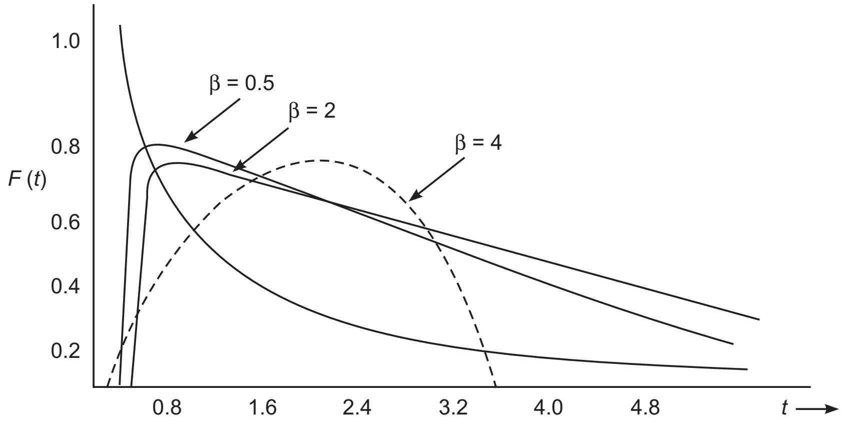


Fig. 52.6: Effect of β on Weibull probability density function.

For $\beta < 1$, the pdf is similar in shape to the exponential and $\beta \geq 3$ [large values], the *pdf* is somewhat symmetrical like normal distribution.

For $1 < \beta < 3$. The density function is skewed. When $\beta = 1$, $\lambda(t)$ is constant and the distribution is identical to exponential.

Theta (θ) is a scale parameter that influences both the mean and dispersion of the distribution. As 'θ' increases the reliability increases at a given point in time. The parameter is also called the characteristic life.

MTTF and variance of the Weibull distribution is given by

$$\text{MTTF} = \theta\Gamma\left(1+\frac{1}{\beta}\right) \text{ and}$$

$$\sigma^2 = \theta^2\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[T\left(1+\frac{1}{\beta}\right)\right]^2\right\}$$

where $\Gamma(x)$ is Gamma function and is given by

$$\Gamma(x) = \int_0^\infty y^{x-1}.e^{-y}dy \quad \text{where } y = \left(\frac{t}{\theta}\right)^\beta$$

52.7.5 Gamma Distribution

The failure density function for gamma distribution is

$$f(t) = \frac{\lambda^\eta}{\Gamma(n)}t^{\eta-1}e^{-\lambda t} \quad t \geq 0, \eta > 0, \lambda > 0$$

where η is a shape parameter and λ is the scale parameter

$$F(t) = \int_0^t \frac{\lambda^n}{\Gamma(n)}t^{n-1}e^{-\lambda t}d\tau$$

If η is an integer, it can be shown by successive integration by parts that

$$F(t) = \sum_{k=n}^{\infty}\frac{\left((\lambda t)^k \exp[-\lambda t]\right)}{k!}$$

Reliability function

$$R(t) = \sum_{k=0}^{\eta-1}\frac{\left((\lambda t)^k \exp[-\lambda t]\right)}{k!}$$

The gamma distribution can be used to model the time the nth failure of the system, if the underlying failure distribution is exponential.

Problem 1: *The reliability of a cutting assembly is given by*

$$R(t) = \begin{cases}(t - t/t_0)^2 & 0 \leq t \leq t_0 \\ 0 & t \geq t_0\end{cases}$$

Determine (*i*) *the failure rate*

(*ii*) *does failure rate increase or decrease with time*

(*iii*) *determine the MTTF.*

Solution:

From the relationship between $f(t)$ and $R(t)$, we have

(*i*) $$f(t) = \frac{d}{dt}R(t)$$

∴ $$f(t) = \frac{-d}{dt}\left(1-\frac{t}{t_0}\right)^2 = \frac{2}{t_0}\left(1-\frac{t}{t_0}\right) 0 \leq t \leq t_0$$

and the failure rate can be determined by

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{2}{t_0(1-t/t_0)} 0 \leq t \leq t_0$$

(*ii*) The failure rate increases from $2/t_0$ at $t = 0$ to ∞ at $t = t_0$.

(*iii*) Mean time to failure

$$\text{MTTF} = \int_0^{\infty} R(t)\,dt$$

$$= \int_0^{\infty} dt\,(1 - t/t_0)^2 = t_0/3$$

Problem 2: *Pdf for a random variable T, the time in operating hours to failure of an engine is given. What is the reliability for 100 hr operating life.*

$$f(t) = \begin{cases} \dfrac{0.001}{(0.001t+1)^2} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

$$R(t) = \int_t^{\infty} f(t)\,dt = \int_t^{\infty} \frac{0.001}{(0.001\,t+1)^2}\,dt$$

$$= \left.\frac{-1}{(0.001\,t^1+1)}\right|_t^{\infty} = \frac{1}{0.001t+1}$$

and

$$f(t) = 1 - R(t) = 1 - \frac{1}{0.001t+1} = \frac{0.001t}{0.001t+1}$$

then,

$$R(100) = \frac{1}{0.1+1} = 0.999.$$

Problem 3: *The probability density function is given by*

$$f(t) = \begin{cases} 0.002\,e^{-0.002t} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

with 't' hours. Determine R (t) and MTTF and also find the median time to failure.

Solution:

$$R(t) = \int_t^{\infty} 0.002\,e^{-0.002t}\,dt$$

$$= e^{-0.002\,t}$$

and

$$\text{MTTF} = \int_0^{\infty} e^{-0.002t}\,dt$$

$$= \left.\frac{-e^{0.002t}}{-0.002}\right|_0^{\infty} = \frac{1}{0.002} = 500 \text{ hrs.}$$

To find the median to failure

Put $R(t_{med}) = e^{-0.002\,t}\text{ med} = 0.5$

Solving for t_{med}, $t_{med} = \dfrac{\ln 0.5}{-0.002} = 346.6$ hrs.

Problem 4: *A particular machine has a constant failure rate of λ = 0.02 hrs.*

(a) What is the probability that it will fail within first 10 hours.

(b) Suppose that the machine has operated successfully operated for 100 hrs, what is the probability that it will fail during the next 10 hours of operation.

Solution: (*a*) Probability of failure for first 10 hours is

$$P\{t \leq 10\} = \int_0^{10} f(t)\,dt = F(t)$$

$$= 1 - e^{-0.02 \times 10} = 0.181$$

(*b*) The conditional probability

$$P\{t \le 110\,|\,, t > 100\} = P\left\{\frac{(t \le 100) \cap (t > 100)}{P\,(t > 100)}\right\}$$

$$= P\left\{\frac{(100 \le t > 100)}{P\{t > 100\}}\right\}$$

$$P\{t \le 100\,|\,, t > 100\} = \int_{100}^{110} \frac{f\,(t)\,dt}{1 - F\,(100)}$$

$$= \int_{100}^{110} \frac{0.02\,e^{-0.02t}\,dt}{1 - 1t\,\exp\,(-0.02 \times 100)}$$

$$= \frac{\exp\,(-0.02 \times 100) - \exp\,(-0.02 \times 100)}{\exp\,(-0.02 \times 100)}$$

$$= 1 - \exp\,(-\,0.02 \times 100) = 0.181$$

Problem 5: *If the two reliability functions have the same mean, show that their reliabilities may be different for the same operating time.*

Solution: Let $R_1\,(t) = e^{-0.002t} \quad t \ge 0$

with $\text{MTTF}_1 = 500$ hrs. [from problem 3]

and $R_2(t) = \dfrac{1000 - t}{1000} \quad 0 \le t \le 1000$

where $\text{MTTF}_2 = \displaystyle\int_0^{1000} \left(1 - \frac{t}{1000}\right) dt = t - \frac{t^2}{2000}\Bigg|_0^{1000} = 500$ hrs.

By computing reliabilities for an operating time of 400 hrs. we obtain,

$$R_1\,(400) = e^{-0.002\,(400)} = 0.449$$

and $R_2\,(400) = \dfrac{1000 - 400}{1000} = 0.60$

Thus, though the mean for the two functions is same, reliabilities are different.

Problem 6: *A linear hazard function λ (t) = 5 × 10⁻⁶ t, where 't' is measured in operating hours. If the reliability of 0.98 is desired, what is the design life?*

Solution:

$$R(t) = \exp\left[-\int_0^t 5 \times 10^{-6}\,t^1\,dt^1\right]$$

$$= \exp\,[-2.5 \times 10^{-6}\,t^2]$$

$$= 0.98$$

$$t_{0.98} = \sqrt{\frac{\ln 0.98}{-2.5 \times 10^{-6}}}$$

$$= 89.89 \approx 90 \text{ hrs.}$$

Problem 7: *A device has a decreasing failure rate characterized by two parameter Weibull distribution with a wear out linear hazard function*

$$\lambda\,(t) = \frac{2}{1000}\left(\frac{t}{1000}\right) = 2 \times 10^{-6}\,t$$

The shape parameter β = 2 and scale parameter, θ = 1000. The device is required to have a design life reliability of 0.99. Determine the design life and MTTF.

Solution: $R(t) = e^{-(t/1000)^2} = 0.99$

The design life is given by

$$t_{\text{Design}} = 1000\sqrt{-\ln 0.99} = 100.25 \text{ hrs.}$$

$$\text{MTTF} = 1000\Gamma\left(1+\frac{1}{2}\right) = 886.23 \text{ hrs.}$$

and

$$\sigma^2 = 10^6\left\{\Gamma(1+1) - \left[\Gamma\left(1+\frac{1}{2}\right)\right]^2\right\}$$

$$= 214601.7$$

or

$$\sigma = 463.25 \text{ hrs.}$$

where $\Gamma\left(1+\frac{1}{2}\right)$ = 0.886227 from the table of Gamma function.

52.8 SYSTEM RELIABILITY

It is always difficult to estimate the reliability of the system comprising of many elements. One approach for analyzing such systems is to decompose the system in to subsystems who's individual reliability factors can be estimated or determined. Depending upon the manner in which these subsystems are connected to constitute the given system, the combinational rules of probability are applied to obtain the system reliability.

The following types of systems are analyzed

1. Systems with components in series
2. Systems with components in parallel
3. Combination of series and parallel systems.

52.8.1 Systems with Components in Series

A series system is represented in the Fig. 52.7. System generally consists of large number of components connected in series. The successful operation of the system depends on the proper operation of all the components *i.e.,* if one of these components fails, the system fails.

In terms of survival, the system can be no better than a component with the lowest probability of survival.

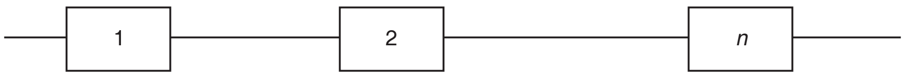


Fig. 52.7: Reliability of System in Series.

The reliability of the system (Rs) can be determined in the following manner.

Let E_1 = the event that component 1 does not fail

E_2 = the event that component 2 does not fail

Then, $P(E_1) = R_1$ and $P(E_2) = R_2$

$\therefore$ Rs $= P(E_1 \cap E_2) = P(E_1) \,.\, P(E_2) = R_1 \,.\, R_2$ assuming that the two components are independent, *i.e.,* failure of one component does not change the reliability of other component, *i.e.,* in order for the system to function, both components 1 and 2 must function.

For 'n' mutually independent components in series.

$$Rs\ (t) = R_1(t)\ .\ R_2\ (t)\ .\ R_3\ (t)\ ....\ R_n\ (t)$$

In this case, the system reliability decreases rapidly as the number of series components increases and the reliability will always be less than or equal to the least reliable component.

Thus, for a series system,

$$Rs \le \min \{R_i\}$$

For example, a system having three subsystems with reliability of 0.6, 0.9 and 0.8 will have reliability.

$$Rs = 0.6 \times 0.9 \times 0.8 = 0.432$$

This system has a reliability Rs = 0.432 which is much less.

Necessary Level of Subsystem Reliability

Let q = the probability that a subsystem will fail.

Assuming 'q' to be identical for all subsystems

$$Rs = (1 - q)^n$$

For example, if we want Rs = 0.9999 in a given system with twenty components, Then we have,

$$Rs = (1 - q)^n$$

Application of binomial series gives,

$$Rs = 1 + n\,(-q)^1 + \frac{n\,(n-1)}{2}(-q)^2 + ... + (-q)^n$$

By ignoring higher order terms assuming that 'q' is small, we have,

$$Rs = 1 - nq$$
$$= 0.9999 = 1 - 20 \times q$$
$$q = 0.0000005.$$

This is a subsystem reliability to meet the requirements of system.

52.8.2 Reliability of a System in Parallel

If two or more components are in parallel (or redundant), the successful operation of any one of the components leads to the successful operation of the system, *i.e.,* any one element fails the system will continue to operate (to function). The block diagram of parallel system is represented in the Fig. 52.8.

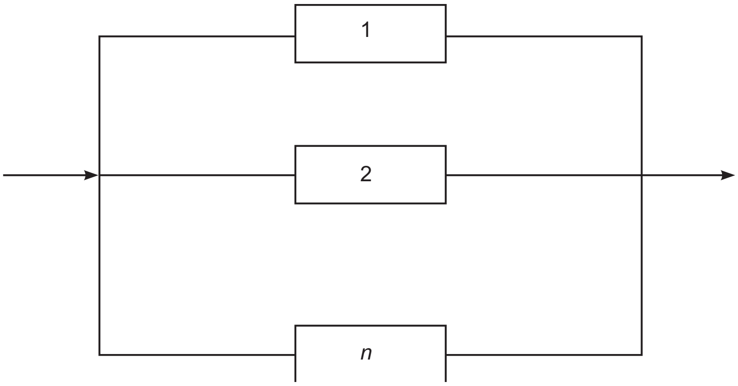


Fig. 52.8: Reliability Block Diagram of Parallel System.

System reliability for n parallel and independent systems is found by taking. One minus the probability that all 'n' components fail (*i.e.,* the probability that at least one component does not fail).

For the two components in parallel,

Consider
$$Rs = P[E_1 \cup E_2] = 1 - [E_1 \cup E_2]^C = 1 - P\left[E_1^C \cap E_2^C\right]$$
$$= 1 - P\left[E_1^C\right].\left[E_2^C\right]$$
$$= 1 - (1 - R_1)(1 - R_2)$$

Generalizing,
$$Rs\,(t) = 1 - \prod_{i=1}^{n}\left[1 - R_i(t)\right]$$

It is always true that,
$$Rs\,(t) \geq \text{Max}\,\{R_1\,(t), R_2\,(t), \ldots, R_n\,(t)\}$$

For example, a system consisting of two elements connected in parallel have their probability of functioning as 0.9 and 0.8. Then, the reliability of the system is,
$$Rs = 1 - [(1 - 0.9) \times (1 - 0.8)]$$
$$= 1 - 0.02 = 0.98.$$

52.8.3 Combined Series Parallel System

Simple combinations of parallel and series subsystems can be easily analyzed by successfully representing subsystems in to equivalent parallel or series components.

Consider a system shown in Fig. 52.9.

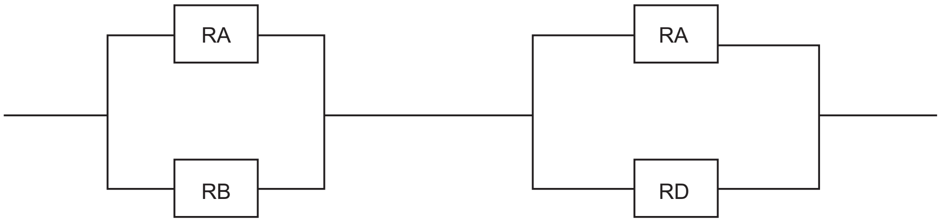


Fig. 52.9: Series Parallel System.

To calculate the reliability of this system, first the system is converted into equivalent series components.

Let $R_A = 0.9, R_B = 0.8, R_C = 0.7$ and $R_D = 0.6$

Then equivalent series component reliabilities are:
$$R_{AB} = 1 - [(1 - R_A)(1 - R_B)] = 1 - 0.1 \times 0.2 = 0.98$$
$$R_{CD} = 1 - [(1 - R_C)(1 - R_D)] = 1 - 0.3 \times 0.4 = 0.88$$

This equivalent series system is represented below

∴ The system reliability is computed as
$$Rs = R_{AB} \,.\, R_{CD}$$
$$= 0.98 \times 0.88$$
$$= 0.8624$$

Consider a parallel series system as shown in Fig. 52.10.

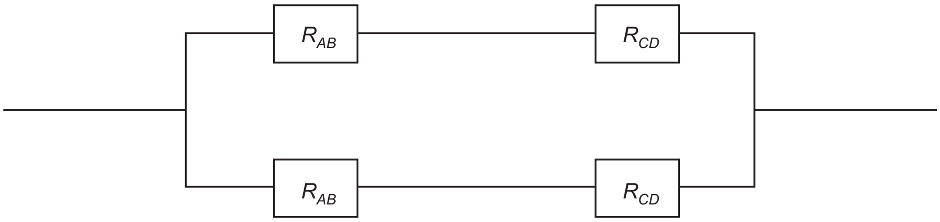


Fig. 52.10: Parallel Series System.

First convert the series subsystems in to equivalent parallel components.

Let $R_A = 0.9, R_B = 0.8, R_C = 0.7$ and $R_D = 0.6$

Equivalent parallel components are:

$$R_{AC} = R_A \times R_C = 0.9 \times 0.7 = 0.63$$
$$R_{BD} = R_B \times R_D = 0.8 \times 0.6 = 0.48$$

System reliability,

$$\begin{aligned} \text{Rs} &= 1 - [1 - R_{AC}]\,[1 - R_{BD}] \\ &= 1 - (0.37) \times (0.52) \\ &= 0.8076. \end{aligned}$$

Now, Consider a system as represented in Fig. 52.11.

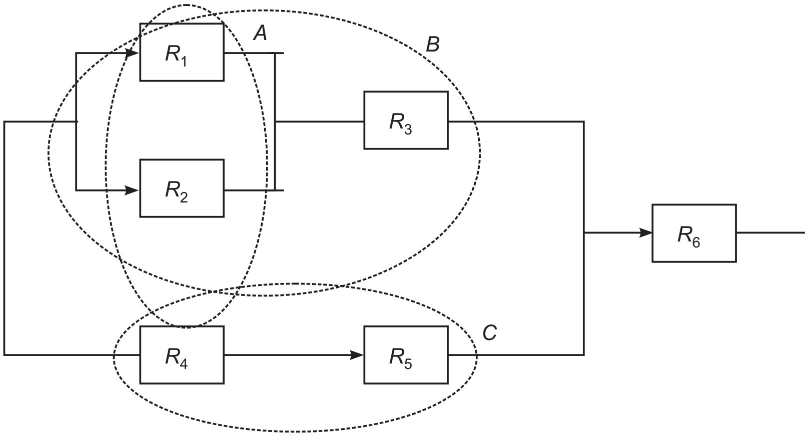


Fig. 52.11: Series Parallel Combination System.

The system reliability is obtained on the basis of relationship among the subsystems. The reliabilities of subsystems are computed as follows.

For a subsystem A,

$$R_A = [1 - (1 - R)\,(1 - R_2)]$$

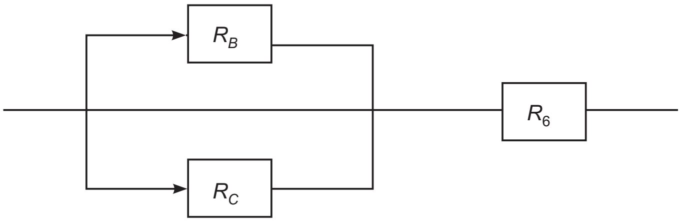
For a subsystem B,

$$\begin{aligned} R_B &= R_A\, R_3 \\ &= [1 - (1 - R_1)\,(1 - R_2)]\,.\,R_3) \end{aligned}$$

and for a subsystem C,

$$R_C = R_4\, R_5$$

The equivalent system is represented as



Since R_B and R_C are in parallel with each other and in series with R_6,

$$Rs = [1 - (1 - R_B)(1 - R_C)] \times R_6$$

Problem 8: *A system is composed of 10 components connected in series. Each component has an exponential time to failure distribution with a constant failure rate of 0.5 per 4000 hours. Compute the reliability of the system for 2000 hours of operation and find MTTF.*

Solution: Component failure rate (λ) = $\frac{0.5}{4000} = 12.5 \times 10^{-6}$ per hour

Reliability of each component after 2000 hours of operation is

$$R = e^{-[(12.5 \times 10^{-6}) \times 2000]}$$
$$= 0.975$$

Reliability of the system after 2000 hours of operation

$$Rs = \exp[-(10 \times 12.5 \times 10^{-6}) \times 2000]$$
$$= 0.779$$
$$\text{MTIF} = \frac{1}{10 \times 12.5 \times 10^{-6}}$$
$$= 8000 \text{ hours.}$$

Problem 9: *For a system composed of three elements in parallel, determine system of reliability for 2000 hrs of operation and find MTTF. The three components have identical failure rate of 0.0005/hr and time to failure distribution is exponential in each case. What is MTTF of each component.*

Solution: Failure rate for each component = 0.0005/hr

System reliability for 2000 hrs of operation is

$$Rs = 1 - [1 - e^{-0.0005 \times 2000}]$$
$$= 1 - (0.63212)^3$$
$$= 0.7474.$$
$$\text{MTTF} = \frac{1}{0.0005}\left[1 + \frac{1}{2} + \frac{1}{3}\right] = 3666.67 \text{ hrs.}$$

MTTF for each component is

$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{0.0005} = 2000 \text{ hrs.}$$

52.8.4 Reliability of k-out of 'm' System

There is another important practical system, one where more than one of its components are required to meet the demands. For example, Two out of four generators are required to supply the required power to the suppliers.

A generalization of '*n*' parallel components occurs when a requirement exists. for '*k*' out of '*n*' components, which are identical and independent to function for the system to function. If $k = I$, complete redundancy occurs and if $k = n$, the 'n' components are, in effect in series. The reliability can be obtained from the binomial probability distribution.

If 'R' is the reliability of each component which are independent. If p is the probability of success of each component, then we have

$$P(x) = \begin{bmatrix} n \\ x \end{bmatrix} R^x (1 - R)^{n-x}$$

This is the probability of exactly x components operating. This is true since,

$$\begin{pmatrix} n \\ x \end{pmatrix} \frac{n!}{x!(n-x)!}$$

is the number of ways (arrangements) in which successes can be obtained from 'n' components.

$R^x (1 - R)^{n-x}$ is the probability of x successes and $n - x$ failures for a single arrangement of successes and failures.

$\therefore \quad Rs = \sum_{x=k}^{n} P(x)$ is the probability of k or successes from among the 'n' components.

52.8.5 Complex System (Non Series Parallel System)

In practice, the systems are not always simple series, parallel systems. A complex system on simplification may produce a non series parallel configuration. A simple structure of structure of non-series parallel structure is represented in the Fig. 52.12.

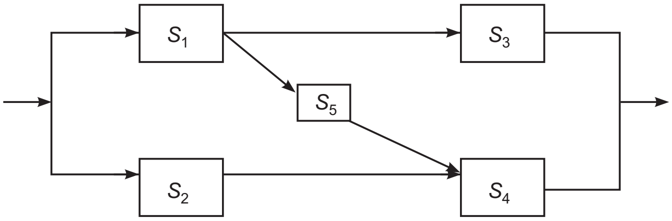


Fig. 52.12: A Bridge Configuration.

The logic diagram approach converts the system diagram into a logic diagram, which consists of simple parallel paths between input and output terminals. Each path contains elements whose successful operation can lead to the success of the system. For successful operation of the system, there should be at least one continuous path between in and out terminals.

The logic diagram for Fig. 52.12 is shown in Fig. 52.13.

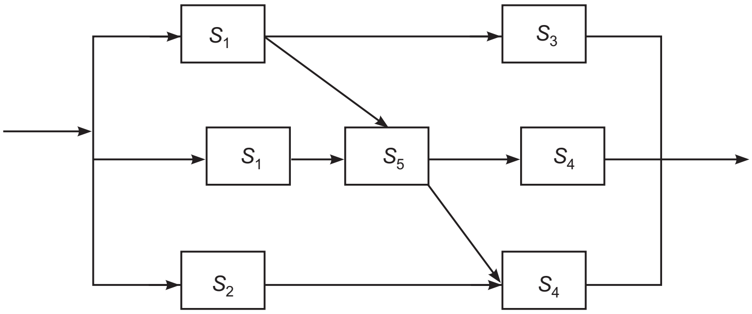
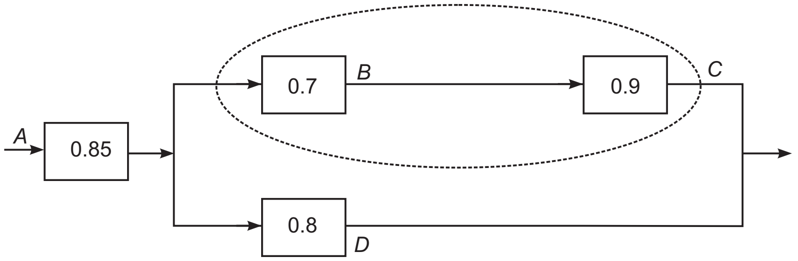


Fig. 52.13: Logic Diagram for Fig. 52.12.

It may be noted that the subsystem S_5 is unidirectional like other subsystem.

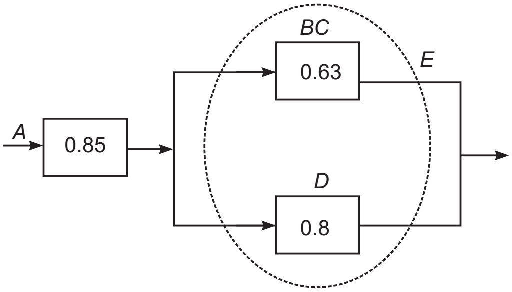
Problem 10: *Calculate the system reliability for units connected as shown below.*



Solution: First, the reliability of system B and C is determined.

$$R_{BC} = R_B \times R_C = 0.7 \times 0.9 = 0.63$$

This is represented in the block diagram below.



Secondly, BC and D are then replaced by E as B & D are parallel.

$$RE = 1 - [(1 - 0.63)\ (1 - 0.8)]$$
$$= 0.926.$$

The system now reduces to simple series system,

System reliability $\quad \text{Rs} = R_A \times R_E$

$$= 0.85 \times 0.926 = 0.7870.$$

52.8.6 Systems with Components in Series (Exponential Model)

Suppose that the system has 'n' components in series, each with exponentially distributed time to failure with failure rates $\lambda_1, \lambda_2, ..., \lambda_n$. The system reliability is given by

$$\text{Rs} = e^{-\lambda_1 t} \times e^{-\lambda_2 t}, ..., \times e^{-\lambda_n t}$$
$$= \exp\left[-\left(\sum_{i=1}^{n} \lambda_i\right) t\right]$$

The mean time to failure is given by

$$\text{MTTF} = \frac{1}{\sum_{i=1}^{n} \lambda_i}$$

If all the components in series have identical failure rates (λ).

$$\text{MTTF} = \frac{1}{n\lambda}$$

52.8.7 Systems with Components in Parallel (Exponential Model)

The system reliability is given by

$$\text{Rs} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

$$= 1 - \prod_{i=1}^{n} (1 - e^{-\lambda_i t})$$

As a special case, if all the components have the same failure rates (λ), system reliability is given by

$$\text{Rs} = 1 - \left(1 - e^{-\lambda_i t}\right)^n$$

$$\text{MTTF} = \frac{1}{\lambda}\left[1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right].$$

52.9 RELIABILITY IMPROVEMENT

A high degree of reliability is an absolute necessity for complex and modern systems to be used for industrial, military and other scientific purposes. There are many ways by which reliability of a component or system can be enhanced. These methods are discussed below.

1. Design and Safety Factor

In order to design reliability into products, reasons for product failures must be analyzed thoroughly. Generally, a product fails prematurely because of inadequate design features, manufacturing and part defects, abnormal stresses induced, environmental condition and human error. Several methods are available for the design engineers to accomplish the requirements. Higher reliability could be achieved through mature design.

Higher safety factors are being used in those cases where there is a doubt regarding , the ability of the certain structure/component to withstand a particular load. Nowadays, exhaustive testing methods are/ available to achieve required degree of reliability.

2. Parts and Material Selection

Designer has to choose between selecting standard parts and manufactured specialized parts with higher reliability and greater tolerances. The trade off is usually in cost but ease of parts availability, ease of repair, energy requirements, weight and size may also be considerations. The historical databases can assist in determining relative reliabilities among competing parts. Knowledge of material properties and the external stresses the system will experience is important. The material properties of materials such as metals, polymers, ceramics and composites include tensile strength, hardness, impact strength, fatigue life and creep.

3. Redundancy

When it is not possible either to manufacture a highly reliable component or the cost associated with such manufacturing is too high, the system reliability can be improved by the techniques of introducing redundancy. This involves the creation of additional parallel paths in the system. Generally, there are two types of redundancies — parallel and stand by to improve system reliability.

In a system of complex nature, redundancy can be applied at various levels.

The various approaches for introducing redundancy in the system are:

(*a*) To provide a duplicate or an additional path for the entire system itself. This is known as system or unit redundancy.

(*b*) To provide redundant path for each component individually which is called component redundancy.

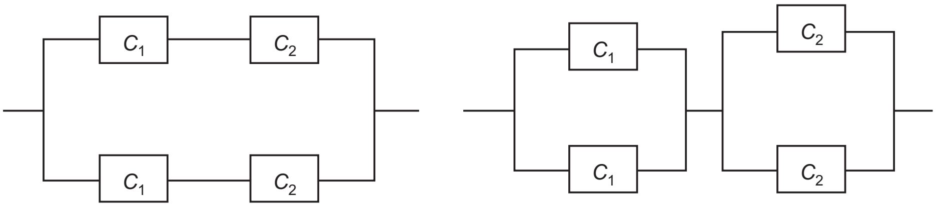
(*c*) Weak component should be identified and strengthened by reliability.

(*d*) Use a combination of the above methods depending upon the configuration called mixed redundancy.

Component versus unit redundancy

It is easier to introduce redundancy at unit level rather than at component level. However, component level redundancy provides higher reliability than unit redundancy.

The two ways of applying redundancy is shown in Fig. 52.14.



(*a*) Unit redundancy (*b*) Component redundancy

Fig. 52.14: Redundancy in Systems.

For unit redundancy, the reliability of the system.

$$Rs_U = 1 - [(1 - R_1R_2)(1 - R_1R_2)]$$

where R_1 and R_2 are the reliabilities of components of components C_1 and C_2.

For the case of component redundancy, the reliability is

$$Rs_C = 1 - [(1 - R_1)^2][1 - (1 - R_2)^2]$$

Suppose, if $R_1 = R_2$, then

$$Rs_U = 2R^2 - R^4$$

$$Rs_C = R^2(2 - R)^2$$

The difference between Rs_C and Rs_U is

$$Rs_C - Rs_U = 2R^2(1 - R)^2$$

i.e., $$Rs_C - Rs_U > 0$$

i.e., the redundancy at the component level is better than redundancy at the unit level as far as reliability is concerned.

Parallel redundancy also referred to as "hot redundancy" is generally resorted to when the individual unit doesn't have the required reliability. If the unit is highly critical or strategic, then more than one unit can be put in parallel redundancy to compensate for unreliability. A compromise is to be struck between the improvement in reliability and the cost of each additional unit.

Stand by redundancy

In stand by redundancy the units are duplicated but one or more units more remain idle (called secondary unit) until the primary unit fails. In standby redundancy the failed equipment or unit is replaced manually or automatically by its equivalent and in such cases the reliability of the operator or sensing and switching mechanism is to be taken into consideration.

Comparison between parallel (redundancy) system and stand by system is shown in the Fig. 52.15.

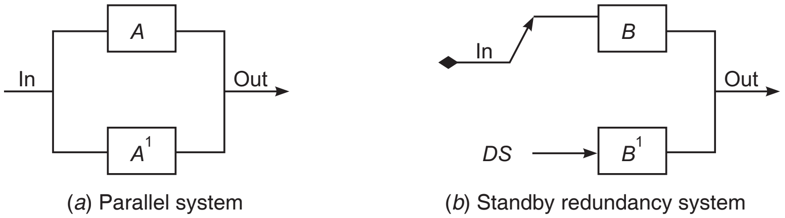


Fig. 52.15: Comparison between parallel and stand by redundancy.

In parallel system, all the paths are active from the beginning of the operation of the system till both of its elements fail. But in standby redundancy, all the paths are not in operation since starting, but will be operating when the first system fails.

In Fig. 52.15, when the system starts operating, decision switch (*DS*) connects the input to the element *B*, where as the other element B^1 is in reserve in a "Off" condition. In case element '*B*' fails, the decision switch senses this by a built in mechanism and the connection is made to the stand by element B^1.

As per the Fig.,

The reliability of the parallel system,

$$Rp = 1 - [(1 - p)\ (1 - p)^1$$

Reliability for the stand by system.

$$\text{Rs} = 1 - p\,(B) \times p\left(\frac{B^1}{B}\right)$$

If both the elements are identical, and reliability of each element is '*p*'. Then reliability of the system is

$$\text{Rs} = p\,(1 - \log_e p)$$

4. Marginal Testing

Marginal testing is prescribed by the designer as a method of predicting probability of failure due to degradation. Marginal testing involves periodic testing on a programmed basis. This helps to isolate degraded parts or components and are replaced before the actual failure occurs.

5. Derating

Derating consists of using a component under stress significantly below its rated value. This is proved to be more beneficial when applied to electronic components, in which case the designed voltage or current strength of the part is well above the normal operating level. Derating curves are provided in military handbook — reliability prediction of electronic components.

6. Quality Control and Z-D Programme

7. Maintainability

Maintainability is used to provide high effective reliability. If parts are readily interchangeable and replaceable, failures can be repaired at a faster rate by replacing defective parts with operating spares. This increases the available of the system.

52.10 MAINTAINABILITY

Goldman and Slattery (1967) has given the quantitative definition of maintainability. Maintainability is a characteristic of design and installation, which is expressed as the probability that an item will be restored to specified conditions within a given period of time when maintenance action is performed in accordance with prescribed procedures and resources.

Maintainability can be expressed as

$$M = 1 - e^{-t/\text{MTTR}}$$
$$= 1 - \exp[-t/\text{MTTR}]$$

where 't' is the specified time to repair and MTTR is the mean time to repair. Thus, maintainability refers to the ease with which preventive and corrective maintainance on a product can be achieved.

Specifications of Maintainability

1. Mean time to repair (MTTR).
2. Median time to repair.
3. Maximum time (Tp) in which a certain percentage of failures must be repaired.
4. Mean System Down Time — It is the average down time including scheduled maintenance but without maintenance delay times.
5. Mean Time to Restore (MTR) — It is the average unscheduled system down time including delays for maintenance and supply resources.
6. Maintenance work hours per operating hour (MH/OH).

Methods to Increase Maintainability

1. Fault isolation and self-diagnostics.
2. Parts standardization and interchangeability.
3. Modularization and accessibility.

52.11 AVAILABILITY

Availability depends on both reliability and maintainability. The availability can be expressed as

$$\text{Availability} = \frac{\text{Up time}}{\text{Up time} + \text{Down time}}$$

"Availability is the probability that a system or component is performing its required functions at a given point in time or over a stated period of time when operated and maintained in a prescribed manner."

Forms of Availability

1. Inherent Availability

The probability that a system, when used under conditions, without consideration of any preventive action in an ideal support situations shall operate satisfactory at a given point of time.

$$A_i = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

Mean time between failures is a factor of reliability and is given by

$$\text{MTBF} = \frac{1}{\lambda} = \frac{1}{\text{Failure rate}}$$

2. Operational Availability

$$A_o = \frac{\text{Mean time between failures (MTBF)}}{\text{MTBF + Mean time waiting for spares + Mean administrative time + Mean time for repairs}}$$

3. Use Availability

$$A_u = \frac{\text{Operation time + Off time}}{\text{Operation time + Off time + Total down time}}$$

52.12 RELIABILITY LIFE TESTING

The basic objective of reliability life testing is to obtain information concerning failures in order to express reliability quantitatively and to ascertain whether reliability and safety goals are met and to improve product reliability.

The factors that are to be considered before any reliability test is being carried out are

- Objective of the test.
- Types of the test to be performed.
- Operating and environmental conditions.
- Number of units to be tested (Sample size).
- Duration of the test.

The various types of tests are described below:

1. Burn-in Testing

This test is carried out to eliminate or reduce infant morality failures by accumulating initial equipment hours and resulting failure before user acceptance.

Primary objective of bum in testing is to increase the mean residual life of components as a result of being survival this test. Items that have failed during the bum in may be discarded and replaced or be repaired. The bum-in testing requires testing of all the units produce for the designated time, so it increases the production lead time and costs.

2. Acceptance Testing and Qualification Testing

This testing demonstrates through life testing that the reliability goals or specifications have been met or determines whether parts or components are within acceptable standards. It demonstrates that the system design meets performance and reliability requirements under specified operating and environmental conditions. Acceptance testing is based on predetermined sample size.

3. Sequential Testing

Sequential testing provides an efficient method for a accepting or rejecting a statistical hypothesis when the sample is highly favourable to one of the two decisions. This test is based on the sequential probability ratio test developed by Wald and is used for reliability and maintainability demonstration or in acceptance or qualification testing.

4. Accelerated Life Testing

This comprises of techniques for reducing the length of the test period by accelerating failures of highly reliable products.

5. Experimental design involves statistical methods that are useful in isolating causes of failures in order to eliminate them.

6. Reliability Growth Testing

The objective of reliability growth testing is to improve reliability over time through changes in product design, in manufacturing processes and procedures. Reliability test and assessments are conducted on prototypes to determine whether reliability goals are being met, if not a failure analysis will determine the high failure modes and the corresponding fixes. The failure modes are eliminated through engineering redesign and the cycle is repeated.

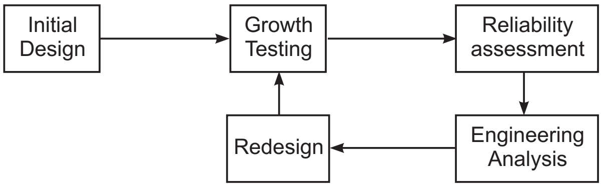


Fig. 52.16: Reliability Growth Cycle.

SUMMARY

Customers always want that the products that are purchased should have a long service life and during this life it should give intended service and utility with few failures. As product becomes more complex, the problems of failures will increase over time. The improvement in effectiveness of such complex systems has therefore acquired special importance in recent years. The effectiveness of a system is its suitability for the fulfillment of the intended function and the efficiency of utilising the means put in to it. The suitability of performing definite task is primarily determined by the reliability and quality of the system. An evaluation of a system reliability becomes essential to decide whether a system will accomplish its mission successfully.

During the past, very high safety factors were introduced which added tremendously to its weight as well as the cost and another approach was to learn from the previous mistakes (failures) of previous designs. These approaches become impractical for the design of new products and systems where each design is different from other and each design has to be right first time and all the time.. Reliability is to be built in to the product right at the design stage and quality and hence the reliability should be built during subsequent stages of manufacture.

Reliability is the ability of an item to perform its intended function under stated operation conditions for a given period of time."Failure of an item represents unreliability. Thus, to compute the reliability of an item, it is necessary to understand the concept of failure. A deviation in the properties of an item from the prescribed conditions is considered as fault. A state of the fault is denoted as Failure."

An item may fail in many ways. An understanding of these failures help in taking appropriate corrective measures for achieving better reliability. The different modes of failure are: **Catastropic Failures, Degradation (Creeping) Failures, Independent Failures**

A high degree of reliability is an absolute necessity for complex and modern systems to be used for industrial, military and other scientific purposes. There are many ways by which reliability of a component or system can be enhanced.

1. Design and Safety Factor
2. Parts and Material Selection
3. Redundancy
4. Derating
5. Quality Control and Z-D Programmes

Availability is the probability that a system or component is performing its required functions at a given point in time or over a stated period of time when operated and maintained in a prescribed manner.

The basic objective of reliability life testing is to obtain information concerning failures in order to express reliability quantitatively and to ascertain whether reliability and safety goals are met and to improve product reliability.

The factors that are to be considered before any reliability test is being carried out are

The various types of tests are Burn-In Testing: Sequential Testing. Accelerated Life Testing Life Tests, Experimental design and Reliability Growth Testing.

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